Equivalent and compensating variation with a max Function

Given the following utility function

$$U(x_1, x_2) = \max\{x_1, x_2\}$$

and with initial prices and income $p_1 = 1$, $p_2 = 5$, and $m = 120$

(a) Find the compensating and equivalent variations if the price of good 1 increases to 3

(b) Graph these variations in a system where the axes are p_1 and x_1

Solution

With $U(x_1, x_2) = \max\{x_1, x_2\}$, the consumer spends everything on the cheapest good. With $p^0 = (1, 5)$ and $m = 120$, the consumer chooses $x_1^0 = \frac{m}{p_1^0} = 120$ and $x_2^0 = 0$, so

$$u^0 = \max\{120, 0\} = 120$$

After the price increase to $p_1^1 = 3$ (with $p_2 = 5$), good 1 is still the cheapest, and the consumer chooses $x_1^1 = \frac{m}{p_1^1} = 40$, $x_2^1 = 0$, so

$$u^1 = \max\{40, 0\} = 40$$

The minimum cost to reach utility u at prices (p_1, p_2) is

$$e(p_1, p_2, u) = u \cdot \min\{p_1, p_2\}$$

The Hicksian demand for good 1 at utility u is

$$h_1(p_1, p_2, u) = \begin{cases} u & \text{if } p_1 < p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases}$$

(a) Compensating and equivalent variation

Compensating variation (CV)

$$CV = e((1, 5), u^0) - e((3, 5), u^0) = 120 \cdot 1 - 120 \cdot 3 = -240$$

Equivalent variation (EV)

$$EV = e((1, 5), u^1) - e((3, 5), u^1) = 40 \cdot 1 - 40 \cdot 3 = -80$$

It should be noted that sometimes the signs of the CV and EV are switched, making one negative or positive. However, their definition in absolute terms remains the same.

The variations can also be calculated without the Hicksian demands or the minimum cost function. The idea is that the equivalent variation shows the amount of money that must be taken from the individual to leave them as well off as they are after the price change. Meanwhile, the compensating variation is the money that must be given to the individual so that their utility remains the same after the price change:

Equivalent variation

Define m' as the income such that, at initial prices, the individual has utility u^1

$$\frac{m'}{p_1^0} = u^1$$

$$m' = 40$$

Then

$$EV = m' - m = 40 - 120 = -80$$

Compensating variation

Define m' as the income such that, at new prices, the individual recovers u^0

$$\frac{m'}{p_1^1} = u^0$$

$$m' = 360$$

Then

$$CV = m - m' = 120 - 360 = -240$$

The interpretation of these values is as follows. For the individual to experience a utility reduction like the one caused by the price increase, it is necessary to take 80 from their income (equivalent variation). On the other hand, for the individual to maintain their utility after the price change, it is necessary to give them an extra 240 (compensating variation).

(b) Graph in the (p_1, x_1) plane

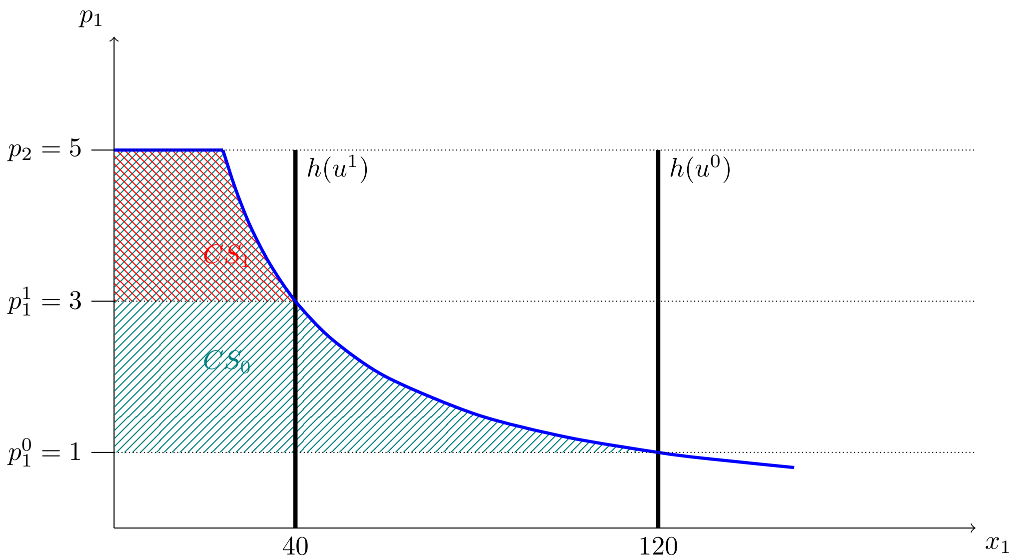
For fixed utility levels, the Hicksian demand is vertical for $p_1 < p_2$. The areas between $p_1 = 1$ and $p_1 = 3$ under $h_1(\cdot, u^0)$ and $h_1(\cdot, u^1)$ represent $|CV|$ and $|EV|$, respectively.

Equivalent integral observation

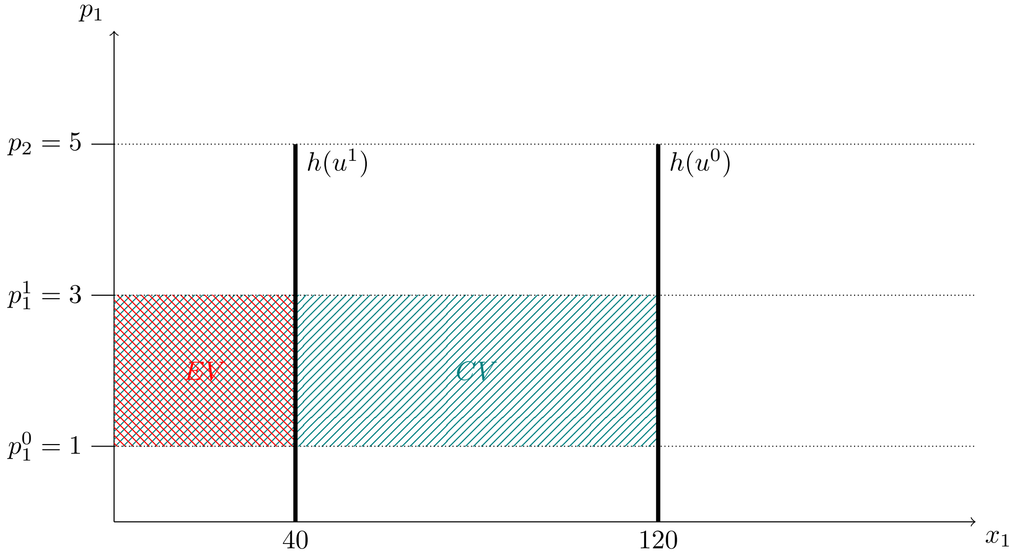
$$|CV| = \int_{p_1=1}^{3} h_1(p_1, 5, u^0)\, dp_1 = 120 \cdot (3 - 1) = 240$$

$$|EV| = \int_{p_1=1}^{3} h_1(p_1, 5, u^1)\, dp_1 = 40 \cdot (3 - 1) = 80$$

In the following graph, we can see the consumer surpluses in each case and the Hicksian demands.



The following graph shows the compensating and equivalent variations based on the Hicksian demands.


p_1
$p_2 = 5$
$p_1^1 = 3$
$p_1^0 = 1$
$h(u^1)$
$h(u^0)$
EV
CV
40
120
x_1